the direction $\mathbf{n} = \mathbf{k}/k$; τ^*, radiation deactivation time of molecules; k, mean absorption coefficient; R^*, probability of radiation deactivation of the molecules.

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METHODS FOR CALCULATING THE ANISOTROPY OF RADIATION BASED ON AN APPROXIMATION OF THE RADIATION PROPERTIES OF SURFACES

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The spatial distribution of radiation is examined in calculating radiation heat transfer between surfaces in a diathermal medium.

Contemporary technological processes require a more detailed study of the spatial distribution of radiation in calculating radiation heat transfer between surfaces in a diathermal medium.

Radiation heat transfer for surfaces with arbitrary emissivities and reflectivities was analyzed very completely in [1] by assuming that the temperature distribution and the optical parameters are given and using the integral equation

$$I_{\text{eff}}(M, s_M) = \varepsilon(M, s_M) I_0(M) + \int_F r(M, s_M, s_{NM}) I_{\text{eff}}(N, s_{NM}) K(M, N) dF_N, \tag{1}$$

where I_{eff} and I_0 are, respectively, the effective and blackbody radiation intensities; $\mathbf{s}$, direction of emission (reflection); r, brightness coefficient; ε, directional emissivity; $K(M, N) = d\varphi(M, N)/dF_N$, where $d\varphi$ is the elementary angular coefficient and I_{eff} is the quantity sought.

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Equation (1) gives an exact description of radiation heat transfer between real surfaces, but it is difficult to apply in engineering calculations since the theoretical and experimental study of the brightness coefficient and the emissivity as functions of direction is a considerably more complex problem than the investigation of hemispherical characteristics of emission and reflection. At the present time, the spatial distribution of energy in emission and reflection can be predicted by electromagnetic theory only for certain optically smooth surfaces [2]. For many engineering materials the brightness coefficient does not vary smoothly with the angles of incidence and reflection. The presence of peaks in the range of small solid angles severely complicates the experimental investigation of the radiation properties of surfaces. This is apparently the reason why some experimenters failed to note the so-called "off-specular" peaks in determining the reflectivities of materials [2]. It was pointed out in [3] that in a detailed experimental investigation of the reflectivity for a single sample of the surface of a material and a single angle of incidence it is desirable to obtain about 20,000 experimental points.

On the other hand, even if the emissivity and reflectivity are known, the solution of Eq. (1) represents a complicated computational problem. Thus, if the spatial distribution of radiation is approximated by 50 zones, a system of 2500 linear algebraic equations must be solved. The problem is still more complicated if the emissivity and reflectivity are given in tabular form.

Thus, the problem of taking account of the anisotropy of radiation is complex and must be solved by the methods of optics, thermophysical experiment, and computational mathematics. It should be noted that it may turn out that mathematical models of surface emission and reflection which are acceptable from the point of view of the description of optical properties may not be suitable for the numerical solution of Eq. (1), while mathematical models of radiation properties of surfaces which are convenient for numerical methods may not describe the emission and reflection mechanism correctly.

Mathematical Models of the Reflection and Emission of Surfaces. In general, the reflection mechanism is determined by reflection from the surface itself and by the scattering of radiation from layers beneath the surface. Generally, emission also is determined not only by the surface layer but also by internal layers. A summary of methods for calculating radiation properties of surfaces when only a thin surface layer emits and reflects is given in [4, 5]. The experimental data confirm the electromagnetic theory predictions for certain clean, optically smooth materials, but for most such materials encountered in practice these theoretical predictions are not in good agreement with experiment.

Since optical properties are very dependent on surface geometry, and since commercial materials are more or less rough, the present trend is to formulate mathematical models which take account of surface topography. Torrance and Sparrow [6] proposed to model a rough surface by a system of mirrorlike facets which reflect according to the laws of geometrical optics. This model can predict the presence of off-specular peaks. In [7] reflection from a rough surface was modeled on a computer by the Monte Carlo method. It was noted in [7] that an adequate description of surface geometry requires a measurement of the rms distance between roughness elements as well as the rms value of the roughness. In addition, the relative fraction of the flat part of the surface is important. As in [6] it was assumed that the roughness elements reflect specularly. In [7] experimental data were compared with calculations in which the rms distance between roughness elements was determined by trial and error. Agreement was obtained for a number of cases.

The emissivity of clean optically smooth materials can be determined by electromagnetic theory. Experiment and calculations agree for some materials [5, 8, 9]. There is a mathematical model of spectral directional radiation for rough metal surfaces [10].

It should be noted that surfaces can be made with prescribed radiation properties to control radiation heat transfer. Thus, in [11] theoretical and experimental analyses were made of the directional radiation properties of V-groove cavities with specularly reflecting walls and perfectly black bottoms. It was shown that such grooves can ensure the emission and absorption of radiation in strictly definite directions. Data in [12] can be used to make artificially roughened surfaces by "dead-end" drilling. The problem was treated for cylindrical cavities with conical bottoms under the assumption that the surface of a cavity radiates and reflects diffusely. Since the grooves and cavities are of appreciable size it is necessary to consider the possibility of temperature variations over the depth of a groove or cavity in artificially roughened surfaces. In this case it is expedient to use optical-geometrical functions.

Taking Account of the Anisotropy of Radiation. Because of the complexity of the solution of integral equation (1), it is common practice to employ mathematical models which either treat both emission and reflection as diffuse, or emission as diffuse and reflection as specular. It has been assumed that the real case lies between these two, but it was shown in [8] that these cases are not limiting, and predictions made with the first and second models can differ from the experimental values by more than 50%.

1. Anisotropic-Diffuse-Anisotropic Model. This model takes account of the anisotropy of self-radiation, and then assumes that all reflections except the last are diffuse. In the last reflection the anisotropy of the radiation is again taken into account. It is assumed that in all reflections except the last the estimate

$$r^{d}_{\min}(M) < r(M,\ s_M,\ s_{NM}) < r^{d}_{\max}(M). \tag{2}$$

is valid. This estimate is convenient when the reflection is not too different from diffuse. From now on we omit the subscript on $r^d(M)$.

It should be noted that the estimate (2) assumes that Kirchhoff's law is violated. As a result of the transformation of Eq. (1) we have the following system of equations (details of the derivation are given in [13]):

$$I^{d}(M) = I^{d}_{\mathrm{ref},1}(M) + r^{d}(M)\int_F I^{d}(N)\,K(M,\ N)\,dF_N, \tag{3}$$

$$I_{\mathrm{eff}}(M,\ s_M) = I_s(M,\ s_M) + \int_F r(M,\ s_M,\ s_{NM}) \times [I_s(N,\ s_{NM}) + I^{d}(N)]\,K(M,\ N)\,dF_N, \tag{4}$$

where $I^{d}_{\mathrm{ref},1}(M) = r^{d}(M)\int_F I_s(N)\,K(M,\ N)\,dF_N$. Solving (3) and (4) we calculate the required quantity $I_{\mathrm{eff}}(M,\ s_M)$. The value of $I^d(M)$ can also be expressed in terms of the resolvent:

$$I^{d}(M) = I^{d}_{\mathrm{ref},1}(M) + r^{d}(M)\int_F I^{d}_{\mathrm{ref},1}(N)\,\Gamma(M,\ N)\,dF_N, \tag{5}$$

where the resolvent is determined from the corresponding integral equation or by light-modeling methods [14].

2. Quasidiffuse Model. The quasidiffuse model is based on the assumption that reflection is diffuse and the reflectivity itself depends on the direction of incidence of the radiation.

It is expedient to apply this model, e.g., when the angle between plane surfaces is not less than 90°. Then multiple reflections are almost completely determined by the diffuse component of the reflection from the surface, and the directional component of the radiation escapes into the surrounding space.

According to the quasidiffuse model of reflection, Eq. (1) can be written as

$$I^{d}(M) = I^{d}_{\mathrm{ref},1}(M) + \int_F r^{d}(M,\ s_{NM})\,I^{d}(N)\,K(M,\ N)\,dF_N, \tag{6}$$

$$I_{\mathrm{eff}}(M,\ s_M) = I_s(M,\ s_M) + I^{d}(M), \tag{7}$$

where

$$I^{d}_{\mathrm{ref},1}(M) = \int_F r^{d}(M)\,I_s(N,\ s_{NM})\,K(M,\ N)\,dF_N.$$

In this model the anisotropy of the effective radiation is completely determined by the anisotropy of the self-radiation. If r^d is a step function, it follows from (6) that

$$I^{d}(M) = I^{d}_{\mathrm{ref},1}(M) + \sum_{j=1}^{n(M)} R_j(M)\int_{F_j(M)} I^{d}(N)\,K(M,\ N)\,dF_N. \tag{8}$$

The solution of Eq. (6) can be expressed in terms of the resolvent

$$I^d(M) = I^d_{ref,1}(M) + \int_F r^d(M, s_{NM}) I^d_{ref,1}(N) \Gamma(M, N)\, dF_N .$$

3. Diffuse-Specular Model. In this model it was assumed that reflection from a surface can be written as the sum of diffuse and specular components which are independent of directions, and self-radiation which is diffuse. Then (1) is transformed into

$$I^d(M) = I^d_s(M) + r^d(M) \int_F I^d(N) S_n(M, N)\, dF_N + \Delta_{n+1}, \tag{9}$$

$$I_{eff}(M, s_M) = I^d(M) + R^{sp}(M) I_{eff}(N, s^{sp}_{NM}). \tag{10}$$

It was shown in [13] that both upper and lower bounds can be estimated for Δ_{n+1}. The function $S_n(M, N)$ characterizes the transport of radiation as a result of specular reflections. In using (9) the necessary number of reflections n must be chosen in accord with the accuracy requirements. This does not take account of the well-known diffuse-specular model [15] which was obtained from physical considerations and actually is based on the fact that it is necessary to take account of an infinite number of reflections. In view of this the expression for $S_n(M, N)$ as $n \to \infty$ must be obtained analytically or by light-modeling methods, which is not always possible. The application of Eqs. (9) and (10) permits the use of the diffuse-specular model for systems with complex geometry for a small number of reflections with a subsequent estimate of accuracy.

4. Anisotropic-Specular Model. In contrast with the diffuse-specular model it was assumed that the specular component depends on the angle of incidence of the radiation and the emissivity in accord with Kirchhoff's law is anisotropic. Then

$$I^d(M) = I^d_s(M) + \int_F r^d(M, s_{NM}) [I^d(N) + I^D_s(N, s_{NM})] \frac{\cos\Theta_{MN}}{\pi}\, d\omega_{MN} + \Delta_2, \tag{11}$$

$$I_{eff}(M, s_M) = I^d(M) + I^D_s(M, s_M) + R^{sp}(M, s^{sp}_M) I_{eff}(N, s^{sp}_{NM}), \tag{12}$$

where

$$\Delta_2 = \int_F r^d(M, s_{NM}) R^{sp}(N, s^{sp}_{N_1N}) I_{eff}(N_1, s^{sp}_{N_1N}) \frac{\cos\Theta_{ML_1}}{\pi}\, d\omega_{ML_1} .$$

By using the method of successive substitutions, Eq. (11) can be transformed in such a way that it will characterize not one but n reflections. Both upper and lower bounds of Δ_{n+1} can be estimated, e.g., $I_{eff}(N) \equiv I_0(N)$ and $I_{eff}(N) \equiv I_s(N)$.

5. Model Based on the Decomposition of Eq. (1) into an Equivalent System of Equations. The kernel of Eq. (1) can be written as the sum of kernels

$$r(M, s_M, s_{NM}) K(M, N) = r^d(M) K(M, N) + r^D(M, s_M, s_{NM}) K(M, N). \tag{13}$$

Then we have the equivalent system

$$I^D(M, s_M) = I^D_s(M) + \int_F r^D(M, s_M, s_{NM}) I^D(N, s_{NM}) K(M, N)\, dF_N + \int_F r^D(M, s_M, s_{NM}) I^d(N) K(M, N)\, dF_N, \tag{14}$$

$$I^d(M) = I^d(M) + r^d(M) \int_F I^d(N) K(M, N)\, dF_N + r^d(M) \int_F I^D(N, s_{NM}) K(M, N)\, dF_N. \tag{15}$$

The solution of (15) can be expressed in terms of a resolvent and substituted into (14).

We note that (15) is considerably simpler than the initial Eq. (1), while (14) is equally complex. However, if the diffuse component of the radiation is large, (14) can be solved with a lower accuracy than (15), but with a negligible effect on the quantity sought:

$$I_{eff}(M, s_M) = I^D(M, s_M) + I^d(M).$$

In conclusion, a number of trends should be noted in the development of methods of calculating radiation heat transfer in a system of real surfaces.

At the present time there is a trend toward developing effective methods of calculating radiation properties of surfaces from the point of view of geometrical and physical optics. However, no one model encompasses a sufficiently broad range of materials.

The practice of calculating radiation heat transfer in the presence of anisotropy has a substantial effect on the formulation of mathematical models of the radiation properties of surfaces. Thus, as a result of approximating the emissivity and reflectivity, new integral characteristics depending on a small number of parameters appeared, but there has been a lag in the experimental investigation of these characteristics. Photographic methods of testing the mathematical models of the radiation properties of surfaces are being more and more widely employed since they permit the determination of the directional distribution of radiation in a single experiment [16, 17]. Extensive use is made of the methods of solving ill-posed problems in processing the experimental data [18, 19].

In taking account of the anisotropy of radiation the method of decomposing the initial complicated equation into an equivalent system of simpler equations is promising. In this case almost complete use is made of the mathematical apparatus for calculating radiation heat transfer by diffusely radiating and reflecting surfaces and the corresponding applied programs or diffuse emission. In addition, when the initial equation is decomposed into an equivalent system each equation can be solved by the optimum numerical method, depending on the specific conditions. Thus, if the norm of the kernel is small, iterative methods are very effective, and other equations can be solved by less accurate but simpler methods such as direct methods of the calculus of variations [20]. In addition, light-modeling methods may be effective with this approach.

NOTATION

I, brightness intensity; r, brightness coefficient; F, surface; s, direction of emission; M, N, P, N_i, L_i, points on surface; ω, solid angle; θ, angle between normal to surface and direction of emission; p, reflectivity; Subscripts: eff, effective; s, self; d, diffuse; sp, specular; ref, reflected; D, directional.

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